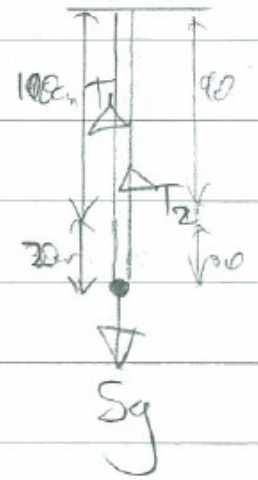
M3 - January 2003

1-



$T_1 = \frac{\lambda x}{a} = \frac{175 \times 20}{100} = 35N$

$5g = T_1 + T_2$

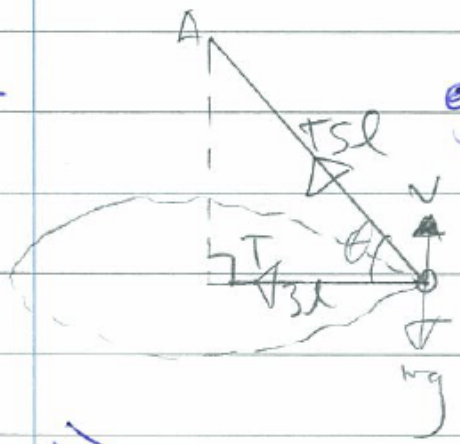
$T_2 = 5 \times 9.8 - 35$

$= 14N$

$14 = \frac{\lambda \times 30}{90}$

$\lambda = 42N$

2-



a) $\downarrow mg = \frac{4}{5} T$

$T = \frac{5mg}{4} N$

b) $\frac{5mg}{4} + \frac{5mg}{4} \times \frac{3}{5} = \frac{mv^2}{3l}$

$2g = \frac{v^2}{3l}$

c) Tensions cannot be assumed to be equal

$$v = \sqrt{6gl} \text{ ms}^{-1}$$

3- a) $\frac{5}{8}r \cdot \frac{2}{3}\pi r^3 \cdot 6^3 + \left(r + \frac{h}{2}\right) \cdot \pi r^2 h = \left(\pi r^2 h + \frac{6^3}{3}\pi r^3\right) d$

$$\frac{5r^2}{2} + hr + \frac{h^2}{2} = \left(h + \frac{4r}{3}\right) d$$

$$5r^2 + hr + h^2 = 2(h + 4r)d$$

$$d = \frac{h^2 + 2hr + 5r^2}{2(h+4r)}$$

b) $d = r$

$$\frac{h^2 + 2hr + 5r^2}{2(h+4r)} = r$$

$$h^2 + 2hr + 5r^2 = 2hr + 8r^2$$

$$h^2 = 3r^2$$

$$h = \sqrt{3}r$$

4. a) $\omega = \frac{2\pi}{T} = \frac{2\pi}{\pi} = 2 \text{ rads}^{-1}$

$v^2 = \omega^2(a^2 - x^2)$

$2.4^2 = 2^2(a^2 - 0.5^2)$

$1.44 = a^2 - 0.25$

$a^2 = 1.69$

$a = 1.3 \text{ m}$.

b) $v_{max} = \omega r = 2 \times 1.3 = 2.6 \text{ ms}^{-1}$

c) $a_{max} = \omega^2 r = 2^2 \times 1.3 = 5.2 \text{ ms}^{-2}$

d) [illegible]

$v = 2.6 \sin 2t$

$2.4 = 2.6 \sin 2t$

$\sin 2t = \frac{12}{13}$

$2t = 1.176, 1.966$

$t = 0.588, 0.983$

∴ Time is $2(0.983 - 0.588) \approx 0.79$s

5 a) $[F = ma]$

$$\frac{60 \times 8000}{(t+2)^2} = 800\frac{dv}{dt}$$

$$60\int_0^t (t+2)^{-2}\,dt = \int_0^v dv$$

$$-60\left[(t+2)^{-1}\right]_0^t = [v]_0^v$$

$$30 - \frac{60}{t+2} = v$$

$$\lim_{t\to\infty} v = 30$$

b) $\frac{dx}{dt} = 30 - \frac{60}{t+2}$

$$\int_0^x dx = \int_0^6 30 - \frac{60}{t+2}\,dt$$

$$x = \left[30t - 2\ln|t+2|\right]_0^6$$

$$= 30(6 - 2\ln 8 + 2\ln 2)$$

$$= 180 - 120\ln 2$$

$$= 96.8\text{m}\ (3\text{sf})$$

6-a) $mgd = \frac{\lambda(d-4)^2}{2a}$

$8 \times 0.5gd = 58.8(d^2 - 8d + 16)$

$4gd = 58.8d^2 - 470.4d + 940.8$

$0 = 6d^2 - 52d + 96$

$0 = (2d - 12)(3d - 8)$

$d = 6 \text{ or } \frac{8}{3}$

But $d > 4 \therefore d = 6m$

b) $\frac{58.5 \times 3^2}{8} = 0.5g \times 3 + \frac{1}{2} \times 0.5v^2$

$\frac{1}{2} \times 529.2 - 6g = v^2$

$v^2 = 205.8$

v_2 [illegible] 14.3 m s^{-1} (3 s.f.)

7 - a) $KE_A = KE_B$

$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mg\left(a + \frac{a}{2}\right)$

$v^2 = u^2 - 3ga$

b) When $u^2 = 6ga$

$v^2 = 6ga - 3ga = 3ga$

[$F = ma$]

$N + \frac{mg}{2} = \frac{mv^2}{a}$

$N = \frac{m}{a}(3ga) - \frac{1}{2}mg$

$= \frac{5}{2}mg$

c) [$F = ma$]

$N + \frac{1}{2}mg = \frac{m}{a}(v^2)$

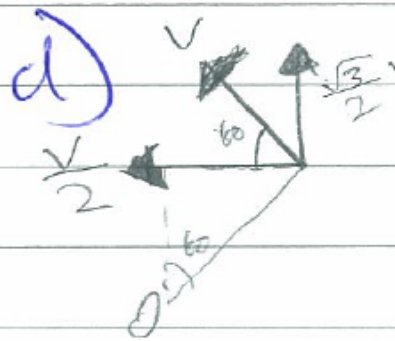
$N = \frac{mu^2}{a} - 3mg - \frac{1}{2}mg = \frac{mu^2}{a} - \frac{7}{2}mg$

$N \geq 0$

$u^2 \geq \frac{7ga}{2}$

$\frac{mu^2}{a} \geq \frac{7}{2}mg$

$u \geq \sqrt{\frac{7ga}{2}}$

d) 

$\uparrow [s = ut + \frac{1}{2}at^2]$

$0 = \frac{\sqrt{3}}{2}vt - 4.9t^2$

$t\left(\frac{\sqrt{3}v}{2} - 4.9t\right) = 0$

$t = 0$ or $\frac{\sqrt{3}v}{g}$

$\rightarrow \frac{1}{2}tv = \sqrt{3}a$

$\frac{\sqrt{3}v^2}{2g} = \sqrt{3}a$

$gv^2 = 2ga$

$v^2 = u^2 - 3ga$

$2ga = u^2 - 3ga$

$u^2 = 5ga$

$u = \sqrt{5ga}$